

Fine-Tuning Arguments and the Concept of Law*

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Abstract Anthropic reasoning based on the apparent fine-tuning of physical parameters – scientific theory’s possession of values in an apparently tiny range allowing life – has been reinvigorated with the realization that string theory, far from determining the parameter values at issue, has models of great diversity. From this fact, many are convinced that the fine-tuning evidence is best explained by the observed universe’s selection as life-permitting (from many real sub-universes most of which do not allow the creation of life). Others see the universe as purposeful and perhaps designed. However, all fine-tuning arguments presuppose a governing conception of laws of nature. This paper argues that a David Lewis-style best-system account of scientific law disarms the anthropic argument. Indeed, in light of the fact that even *rejected* scientific theories are in many cases fine-tuned, anthropic reasoning may point toward a deflationary metaphysics rather than the extravagant designer-or-multiverse alternative.

Physical parameters, from force strength constants to initial conditions of the universe, are often claimed to be “fine-tuned” for life. Small manipulations of these theoretically unconstrained parameters typically lead to models of the universe with no significant complexity and so no life. Such model universes may represent space-times that exist for mere seconds, expand too rapidly for any complexity to result, or include little chemical complexity greater than that of a helium atom. From this fact about hypothetical parameter manipulation, that actual parameters are just right for life, it may seem that the actual universe is as it is *for us*.

Within the scientific community, the main naturalistic explanation for fine-tuning involves the reality of a multiverse of many “pocket” universes – each with its own governing bylaws. Given that the multiverse has the requisite variety of such laws, fine-tuning is a matter of observer selection: Life and observation can arise but only in those sub-universes of the multiverse where bylaws and initial conditions are in the range allowing significant complexity.

It may seem, then, that an explanation of fine-tuning requires the extreme metaphysical measures of teleology, design, or multiverse: “anthropic” explanation. To the contrary, I will argue, there is reason to think that the fine-tuning argument points only toward a rethinking of the law-of-nature concept. The idea is simple. Fine-tuning of scientific theories, fine-tuning for complexity and life, is best understood as *an artifact of theoreticians’ design*.

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Scientific laws, on my proposal, are concise and useful *descriptions* rather than governing principles of nature with life-promoting function. Mathematical modeling of complex systems – from a bacterium to a cosmos – often involves fine-tuning. Moreover, theories now long *rejected* are sometimes fine-tuned. For example, *pre*-inflationary cosmology circa 1980 required fine-tuning for the flatness of space-time. But that theory is now superseded by one not requiring fine-tuning for flatness. It follows, I will argue, that a David Lewis-style best-system account of law is motivated and perhaps confirmed by the fine-tuning. The upshot is that a successful fine-tuning argument may point toward a deflationary metaphysics for laws rather than the extravagant designer-or-multiverse cosmology.

1. Fine-Tuning and Anthropic Explanation in Physics

Two well-known examples of fine-tuning are worth detailing. Pre-inflationary cosmology was deeply concerned with several anthropic coincidences including the “flatness problem” just mentioned. At very large scales the universe is approximately flat (despite the smaller scale gravitational curvature apparent at scales from those of planets to clusters of galaxies). A pre-inflationary cosmological model departing significantly from flatness at early times may involve a rapid collapse (due to excessive energy density) or rapid expansion (due to insufficient energy density); either option plausibly disallows both complexity and life as we know it. Indeed, to allow complexity and life, the model’s early stages needed to have a *very* precisely tuned energy density. Estimates had it that, at the Plank time, the energy density required tuning to one part in as much as 10^{60} if complexity and life is to be possible. Surely, went the thinking of the time, such fine-tuning needs an explanation. And, indeed, inflationary cosmology of the early 80’s was able to explain *away* the need for this inordinate tuning. Exponential stretching of space in the very early universe, inflation, provides a mechanism forcing universes with a broad range of initial energy densities to a flat universe by the end of the inflationary era. Thus, given inflation, no early tuning of energy density is needed or postulated. For about two decades following the introduction of the inflationary model, much of the physics community expected to find that advances in theory obviated the need for anthropic explanation.

In recent years, however, anthropic explanation in physics is resurgent if still deeply controversial. One reason for the resurgence has to do with the fine-tuning of the cosmological constant, the (putative) constant energy density of *virtual particles*. Particles, for example electrons and positrons, are created and quickly destroyed via quantum fluctuations within the vacuum. Strikingly, their vacuum energy may have a negative pressure which accelerates the expansion of space-time. The vacuum energy density of known fields at well-understood energies is

tentatively calculated to have an “absurdly large value, 10^{94} g/cm³.... A[n energy density] of this magnitude would cause the universe to expand with a stupendous acceleration.” (Vilenkin, 2004) However, various theoretical possibilities, conjectured fields and particles, may provide negative energy and positive pressure thus working to cancel some or all of this “absurdly” large value. Now, the fine-tuning problem arises because the cancellation must give a value close to zero g/cm³: Much more and the universe would expand too quickly to allow for life, much less (it’s sometimes also argued) and the universe would have collapsed before we came to be. This problem has seemed particularly significant not only because there is a need for such exact cancellation – by a factor of something like one part in 10^{94} – but also because the calculated value for known fields sets such a wide range of physically plausible vacuum energy densities. It seems that the vacuum energy density truly could be anywhere in this range yet happens to be set just right for life. One may well ask: Why did our universe get so lucky?

Moreover, Steven Weinberg’s calculations in the late 1980’s, postulating humans as “typical” observers, predicted a small but non-zero value for the vacuum energy density. Soon thereafter, evidence from supernova studies provided stunning confirmation of this value. Much of the physics community remains deeply impressed by these results: both the stupendous fine-tuning required for cancellation and the predictive success of anthropic reasoning. Though justifiably controversial, this fine-tuning of the vacuum energy density is in need of either physical *or* philosophical explanation.

A second reason for the resurgence in anthropic explanation comes from string theory’s failure to provide a unique model which, it had been hoped, would determine the fine-tuned parameter values theoretically. Much to the contrary, string theory proponents now explain the fine-tuning in terms of anthropic-multiverse reasoning with a twist from string theory’s many free parameters: There is complexity and life in the observed universe because it is one of the very special parts of an enormous string theory “landscape”, one of the few allowing life. The parameter manipulations, it is argued, take place not just in models but also in “pocket” universes which condense out in the inflationary multiverse. (Within its 11 dimensional space, string theory has the resources – hundreds of free parameters – to construct practically limitless universes with different sorts of governing bylaws and initial conditions; this is the landscape of *possibilities*. Localized quantum fluctuations to these free parameters, it is proposed, *populate* the landscape as the universe undergoes chaotic inflation. See, Weinberg, 2005 and Susskind, 2006.)

2. Fine-Tuning Arguments and Governing Laws

Physicists are usually very clear that fine-tuning is a property of laws that *govern*, i.e., of laws that in some way are responsible for, bring about, or sustain what happens in the universe. For example, Paul Davies writes repeatedly that laws “govern” and also that to meet the requirements to allow life, “certain stringent conditions must be satisfied in the underlying laws of physics that regulate the universe, so stringent in fact that a bio-friendly universe looks like a fix”. (Davies, 2007) The laws, then, are taken as *responsible* for the natural realm they govern, a universe including life. Though speaking for the less spiritual side of the debate in physics, Susskind also repeatedly describes laws that “govern” and, again like Davies, characterizes the fine-tuning argument in terms of laws *responsible* for the nature they govern.

Many coincidences occur in the laws and constants of nature that have no explanation other than “If it were otherwise, intelligent life could not exist.” To some it seems as though the Laws of Physics were chosen, at least in part, to permit our existence. (2006, 21)

As described just above, Susskind *rejects* this argument for intelligent design holding that selection in the multiverse gives the preferred explanation. Here too the assumption of governing laws plays a role: Bylaws of non-life permitting subuniverses govern in such a way that life cannot arise; so (of course) human existence requires or “selects” bylaws capable of bringing about and sustaining life.

Philosophers’ work on fine-tuning also relies on governing laws but usually is cast in probabilistic form. This involves the “likelihood” of hypothetical explanations for fine-tuning. A particular hypothesis has a high likelihood relative to evidence E when it makes E highly probable: the likelihood of hypothesis H on evidence E is just the conditional probability of E given H, $P[E|H]$.¹ Thus, we may say that the presence of a cough has high probability given a cold virus infection, or restating, that the cold virus infection has high *likelihood* given the presence of a cough. Of course, this likelihood does not mean a cold virus infection is probable given a cough; there are other hypotheses with high likelihood relative to a cough, e.g., an allergic reaction. Only when a hypothesis H has high prior probability as well as has high likelihood given evidence E may the evidence E seem compelling for H. Still, the

¹ I suppress background information from the representation of probability claims. $P[E|H]$ is the probability of E given both H and background assumptions.

conditional probability of H given E is greater than the probability of H when the likelihood of H is higher than of \sim H relative to E.²

The fine-tuning argument for theism would seem to be a case in point, especially if for the sake of argument we temporarily set aside the possibility of a multiverse and compare only the hypothesis of *traditional theism* to the hypothesis that *the universe is single and excludes the supernatural*. The fine-tuning argument in question proceeds from anthropic evidence of some sort: that life is permitted, that conscious observers exist, that physical parameters are fine-tuned for life, or that the universe works to allow humans to crack its “code”. Then, advocates of theism sometimes propose, the probability of the anthropic evidence is relatively low given a naturalistic, single universe hypothesis but relatively high given traditional theism. Restated, single-universe naturalism has lower likelihood than theism: $P[\text{anthropic evidence} \mid \text{the naturalistic single universe hypothesis}]$ is low compared to $P[\text{anthropic evidence} \mid \text{traditional theism}]$. If these claims are true, then the anthropic evidence supports traditional theism to some extent as per footnote 2. I will consider several influential analyses of likelihood argument for theism to illustrate how *governing* laws are presupposed.

Elliot Sober (2003) analyzes and rebuts the likelihood claims. He argues that the fine-tuning evidence is a mere artifact of an observer selection effect: we could not very well observe a universe untuned for life. Sober concludes that there is no objective probability (greater than 0) that *our* universe is untuned; thus the likelihood of *both* theism and naturalism is high given the tuning evidence. Sober’s argument is a controversial one; see (Weisberg, 2005) for criticism. Here I only need to point out that Sober understands that the anthropic argument presupposes laws that “govern” and, so, might be the tool of a god “who wanted life to exist and who arranged the Universe so that this would occur” (pp. 44-5).

Another pair of analyses, those of Collins (2003) and Monton (2006), consider *epistemic* probability for life arising in a universe. The details vary between the papers, but put roughly, $P[\text{life is permitted} \mid \text{the naturalistic single universe hypothesis}]$ is to be compared to $P[\text{life is permitted} \mid \text{traditional theism}]$. Fine-tuning may suggest that the latter probability is significantly higher thus giving theism greater epistemic likelihood than single-universe naturalism. However, epistemic probability from the perspective of normal humans allows no such relation of higher probability: $P[\text{life is permitted} \mid X]$ is high for most *any* X because we are very certain that there is life (i.e., very certain that there is no Cartesian demon deluding us on biological reality). This is the problem of old evidence: our

² These claims are supported by simple consequences of Bayes’ theorem. E.g., $P[H|E] > P[H]$ iff $P[E|H] > P[E|\sim H]$. So, a likelihood of H given E greater than that of \sim H given E means that E supports H.

near certainty that life exists hides the supposed implication that life would be improbable in a godless universe (were it anything like ours, i.e., in need of fine-tuning). Both Collins and Monton argue that our embodied perspective must be subtracted out if we are to understand how the evidence that life is permitted rationally reflects on theism. They disagree on just how this must be done and whether theism will then have greater likelihood. Still, for present purposes, it suffices to note an agreement: Their probability judgments are made *presupposing governing laws*. After somehow subtracting out our knowledge that there is in fact life in our universe, the probability of a life permitting universe given the hypothesis of theism is to be judged on the presumption that God may be expected to select laws *leading to life*.

As a final example, consider the work of Richard Swinburne. Swinburne has long given a likelihood argument for God based (in part) on fine-tuned laws that *govern* (Swinburne, 1991). It is his conclusion that only if God is given is there a high probability that *governing* laws are life-permitting and knowable. However, (Swinburne, 2004) shows how his earlier reasoning depends on the *nature* of laws. For example, Swinburne argues that David Lewis's best-system account describes laws that are unable to *explain* events, causation, and the fact that humans can know laws.³ I find this argument very important and have elsewhere tried to show how such explanatory demands may be met by the best-system approach properly formulated (Halpin, 2003). But the next section will press a very different explanatory concern: I will argue that Lewis's account of laws can and should be used to *explain away* the fine-tuning problem.

3. A Deflationary Proposal

Even while contemporary physics is cosmologically inflationary, there is hope for metaphysical deflation. Anthropic explanation is based on the replacement or embellishment of God as *governor* with laws that *govern*. Lewis's empiricist conception of laws dispenses with governing in favor of a systematizing description. But such empiricism about laws is not just a philosopher's conceit. For example, Richard Feynman has sometimes also hinted at such a view:

[I]t is possible to condense the enormous mass of results to a large extent--that is to find laws which summarize.... (Feynman, 1963)

³ For example, "[i]f there were no God, it would be a highly improbable coincidence if events in the world fell into kinds in such ways that the simplest extrapolation from the past frequently yielded correct predictions." (2004, 299).

The best-system account has various formulations but a generic version is sufficient here. Following David Lewis (1973), this empiricist conception of law presupposes a basis of all occurrent fact – i.e., non-counterfactual, non-dispositional, non-nomic fact – including Feynman’s “results”. One is to think of an ideal systematic representation of all such facts; this ideal description is a compromise between the *simplicity* of exposition and *strength* of content. For current purposes, it is sufficient clarification to demand that the simplicity-strength compromise be understood on the example of science as practiced. Finally, then, the best-system account or “BSA” holds that to be a law of nature is just to be a member of the deductive closure of the ideal description. From the perspective of the BSA, then, laws of science are *descriptive rather than governing*: The occurrences in the universe determine the laws of science not vice versa.

There is much debate about the BSA; its tenability will ultimately depend on details of a preferred formulation. I will not defend my version here⁴ but simply try to show how the BSA – even in generic form – undermines fine-tuning arguments. First, because the BSA implies that the laws of nature are descriptions, ideal systematizations but still descriptions of a special sort, the BSA provides no means for a god to control the universe by first designing laws. If indeed there is a designer of the universe, BSA-laws are *consequences* of the totality of events resulting from the design. Laws according to the BSA are *not*, then, the designer’s tool to produce these events. In addition, the BSA-proponent has an alternative explanation for the fine-tuned nature of scientific laws: they *are* designed but *by* theoreticians. So, the BSA may indicate that *the appearance of universe design in the fine-tuning is merely an artifact of human systemizing*.

I believe that this last is the correct conclusion for the BSA proponent to draw. Still, even if physicists are responsible for the tuning of physics laws (rather than God or some sort of observer-selection in a multiverse) one may wonder about the apparent improbability of our universe. Even if the BSA is correct, one might argue, there is still something like one chance in 10^{94} that the vacuum energy density/cosmological constant is correct for life. And, so this counterargument might conclude, even the BSA proponent should be surprised by our own existence. I want to step back to analyze these last claims.

Compare two conditional probabilities starting with the probability that laws are fine-tuned for life, FT, given the governing conception of laws, GL. (I take “GL” to mean that all laws of nature are governing laws.)

(*) P[FT|GL]

⁴ My defense is found in Halpin (2003).

This probability may appear to be low. If so, then the surprising fact of fine-tuning may call for explanation: What accounts for fine-tuning in the governing principles underlying and/or bringing about the universe? Theism and/or a multiverse hypothesis are the only explanations on offer. In any case, if fine-tuning of *governing* laws is to be explained, a metaphysical account would be required.

Now compare (*) to the probability that laws are fine-tuned, given the truth of the BSA.

(**) P[FT|BSA]

This probability too may seem low. But if so, and if fine-tuning of best-system laws requires an explanation, then the account to be given will plausibly be mathematical rather than metaphysical: an explanation of a fine-tuned systematization will be in terms of the tools of human theorizing, *viz.*, mathematical modeling.

Fortunately, there *is* an explanation to be found in mathematical physics giving reason both to expect the fine-tuning of best-system laws in a complex universe and to assign (**) a fairly high probability. In order to consider fine-tuning of universe models, one considers how physical parameters, constant for one model, may vary between models. Here the models in question represent complexes of interacting matter and fields in space-time. So, a systematic account of a universe takes physical parameters *as initial or boundary conditions for the time evolution of a highly complex system*. Such descriptions of complex systems are often highly sensitive to the conditions of their definition. By analogy, think of the standard design description given a skyscraper. Tinker with the building proportions or material strengths and the likely result is collapse. Standard representations of many other complex structures, from computer programs to organic molecules to weather systems, are similarly fine-tuned: small alterations show their instability or sensitivity to initial conditions. Likewise, it is plausible that universe models different from our own in their defining parameters, even slightly different, will often be very different in the time evolution of their structures. Such parameter manipulations may well “break” the complex structures needed for life. Or so I claim. Let me try to further support this plausibility argument.

Complexity is notoriously, and even definitionally, related to intricate interactions between sub-systems and a resulting sensitivity to initial conditions. For example, (Parisi, 2008) gives this definition: “A system is complex if its behavior crucially depends on the details of the system.... [T]he behavior of the system may be extremely sensitive to the form of the equations of motion and a small variation in the equations of motion leads to a large variation in the behavior of the system.” After describing this crucial sensitivity, fine-tuning, in the construction of stable proteins, Parisi gives a close analog to the fine-tuning of universes for life:

At the end we can use this information [on the proteins of E. coli bacteria] to construct a model of the living cell. The model may lead to a system of few thousand (or more) coupled differential equations which can be studied by a lengthy computation on a computer. If the information is accurate enough, the model will describe a living cell. However we know that in real life many mutations are lethal, and therefore it is quite likely that imprecision in the form of the interactions among proteins will lead to a non-living cell or to a living cells with a quite different behavior. (p. 10)

In Parisi's case, imprecise modeling may represent a cell unable to behave normally or even stay alive. Likewise, theorists contemplating scientific laws with slightly varying physical parameters should be prepared for large variation in the results (when these laws are applied to generate whole-universe models in all their complexity). Thus, small variations to physical parameters, mutations so to speak, may likewise be lethal to life. Indeed, this is just what we observe in much universe modeling under the heading of "fine-tuning".

Juhl (2006), "Fine-tuning is not surprising", makes a somewhat similar argument to my own (though *not* meant to be in the service of the BSA):

Suppose ... that some data set corresponding to some physical system S is 'fitted' with some complicated polynomial curve selected from a class of candidate data-fitting curves. A causally ramified phenomenon within system S will likely depend on a fairly wide range of features of the curve, such as the number of maxima and minima, curvatures at various locations, and others. It is likely that many of the constant factors of the polynomial will be highly constrained by the broad subsets of the data that correspond to causally ramified phenomena within such a system. If we have several polynomials that share some variables, or coupled partial differential equations, even greater sensitivity should be fairly typical, if not virtually certain.

But such sensitivity to the precise settings of the constants is just fine-tuning. (p. 272)

I think the best-system theorist should agree and expect fine-tuning in the systematizing accounts (scientific theories) given for the causally ramified universe in which we live.

To this point, I have presented purely theoretical considerations to show that (**), $P[FT|BSA]$, is plausibly high: the complexity of the universe makes fine-tuning probable. Still, these theoretical considerations can be tested. Indeed, I believe there is a test of the BSA in this prediction of fine-tuning. The idea is that if universe models in general should frequently require fine-tuning for life, then even rejected, unsuccessful (and false) models should require such tuning. That is, by the argument of the last paragraphs, the BSA-proponent should expect many models

of the universe to exhibit fine-tuning, even those that ultimately *fail* the test of experiment and are rejected. In fact, numerous now rejected cosmological accounts *are* fine-tuned: Cosmological models of flatness (discussed in section 1) and smoothness in the very early universe, once paradigm cases of fine-tuning, are now rejected in light of the success of inflationary models.⁵ Also, some of the first inflationary models of the universe, models now rejected, require fine-tuning. So, a prediction of the best-system account is confirmed: even rejected, false theories will sometimes be fine-tuned. It is not clear to me just how the proponent of governing laws can make a similar argument to predict fine-tuning.⁶ In any case, neither the God-as-designer view nor the multiverse hypothesis straightforwardly explains the fine-tuning of *false* theories.

An earlier concern will help sum up the BSA on fine-tuning. The cosmological constant/vacuum energy account of the increasing rate of universe expansion may or may not in the end prove correct.⁷ But either way, this constant's tuning to one part in 10^{94} is a matter of mathematical modeling of our complex universe rather than an indication of the improbability of life arising in an arbitrary cosmos. To think otherwise would be to project measures from our best-system (or attempts at a best-system) back onto the world.⁸ The proponent of the BSA will have no justification for this projection.

There is, then, reason to doubt that the fine-tuning argument cogently leads science to significant metaphysics: From the perspective of the BSA, neither a purposeful nor a multiple universe needs to be postulated to account for fine-tuning. Instead, a deflationary account of laws sustains a simpler, naturalistic picture of the world *especially* in light of the evidence of fine-tuning.

⁵ To reiterate, instead of arising with flatness tuned in from the outset, says the consensus view, the universe became flat only from the effects of inflation. So, the universe-tuned-flat-from-the-beginning model is now seen as wrong.

⁶ I have argued that the epistemic probability $P[FT | BSA]$ is high because systematization for time evolution of a complex system like the universe is typically very sensitive to initial conditions. On the other hand, the proponent of governing laws may see science as particularly sensitive to matters that go beyond systematization, e.g., to non-Humean explanatory power or other non-Humean nomic matters. Aristotelian laws or powers, perhaps? For this reason, I think that $P[FT | GL]$ is *not* clearly high.

⁷ This cosmological constant account is yet another fine-tuned theory that may be rejected. See Stenger (2007, pp. 151-3) for the claim that the account may be replaced with a theory of quintessence in need of no fine-tuning.

⁸ Here I am indebted to personal correspondence from Cory Juhl. Also, it should be noted that none of my argument is meant to show that life is probable given the best-system account of laws. Nor, of course, is any of this reasoning regarding the fine-tuning of laws meant to undermine other arguments from design.

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