**(Thanks to Professor Jerrold W. Grossman)**

An application to the Oakland University Summer Mathematics Institute includes a submission of work done on the following set of problems.

Some of these problems are very challenging. The purpose of this is we need to see examples of your problem solving skills at this stage in your mathematical career. You know that sometimes a problem can appear very hard initially, but a day or two later can be solved easily. So allow yourself a lot of time to work on these problems.

You do not need to work on all of the problems. You do not need to solve all of the problems you work on. There is variety here so we can see what you like to try and how well you solve problems. But good solutions and partial solutions on a larger number of problems demonstrates the flexibility and skills we look for in OUSMI participants.

For each problem you do attempt, put your work on separate paper. This should contain your solution if you have one, plus a summary of your work. This summary should contain your thoughts about the problem, partial results, conjectures you considered and why, how you tested your conjectures and the results, and anything else you wish to say. Sending a partial solution, or something you think is wrong with the problem (say so!), and thoughts and ideas about a problem is much better than sending nothing.

Exercise your problem-solving skills. Sometimes it helps to look at similar but simpler problems, or special cases. Sometimes a problem can be given graphical, geometric, or other visual representation which suggests an approach. Keep good, organized data. Be playful and try to help your mind see connections between the special cases, your representations, and the general ideas needed for the actual proble,. Look for patterns. These patterns can sometimes provide insights into how to prove the general case.

You may look at material in libraries or other sources if you wish. Getting specific help from another person is not allowed. All help on these problems from any source should be acknowledged in your comments. We hope you enjoy working on these problems.

The Problems:

1. Do there exist irrational numbers r and s such that rs is rational? Do there exist irrational numbers r and s such that rs is irrational?
2. Does there exist an infinite collection of sets such that the intersection of every two distinct sets in the collection is nonempty, but the intersection of every three distinct sets in the collection is empty?
3. Consider the number 29876543. Determine (1) the number of digits it has; (2) its last three (rightmost) decimal digits; and (3) its first three (leftmost) decimal digits.
4. Find the sum of the 4536 numbers from 1,000 to 10,000 which have all their digits distinct. This is really two problems: Find a solution by using brute force on the computer; find a solution by doing it analytically by hand.
5. Three positive integers a, b and c form a Pythagorean Triple if a2+b2=c2. It is primitive if they have no common factors. For example, 3,4 and 5 form a primitive Pythagorean Triple, 5,12 and 13 form another primitive Pythagorean Triple. Find two more primitive Pythagorean Triples.
6. Suppose your worst enemy gives you n red points and n blue points in the plane, no three of them collinear. (Your enemy picked them, so they are not nicely arranged.) Show that it is possible to draw n line segments, each segment joining a red point to a blue point (so that each red point is paired with a unique blue point, and vice versa) in such a way that none of the line segments intersect.
7. Show that the expressions 2x+3y and 9x+5y are divisible by 17 for the same set of integral values of x and y.
8. A round-robin tournament was held among 13 players. Each player played one game against every other player. Each player won six games. How many triples (K,L,M) of players are there such that K beat L, L beat M, and M beat K?
9. Suppose n children holding loaded water pistols are standing in an open field, no three of them in line, such that all the distances between pairs of them are distinct. At a given signal, each child shoots the child closest to him or her with water. Show that if n is an even number, then it is possible (but not necessarily the case) that every child gets wet. Show that if n is odd, then necessarily at least one child stays dry.
10. How many zeros are there at the end of the number 1000!=(1)(2)(3)...(999)(1000)? How many for 1001!, 1002!, 1003!, 1004!, and 1005!?