Statistics of Leveraged Exchange Traded Funds

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Investment

1. Investment is an aspect of Data Science dealing with trading activities
   - Data used in technical analysis: charts and patterns...
   - Data used in the form of Company's financial health and other related information
   - Qualitative data, such as political news (elections, Clinton's e-mail episode), world events (Brexit), catastrophic events (Fukushima), Social activities (Facebook)....
   - Basic issue is prediction of the behavior of the market.
   - Perhaps one of the most challenging modeling problem with large noisy data.

Simplest Investments

Stocks:
Very much affected by various factors;
Can be traded any time during trading hours or even after hours;
They can be very volatile.

Mutual Funds:
Baskets of stocks;
Effects of various factors is lessened due to averaging;
Can be bought and sold only once a day at the end of the day;
Hence less volatile than individual stocks.

Simplest Investments

Exchange Traded Funds (ETF):

Similar to mutual funds;

Baskets of stocks;

Effects of various factors is lessened due to averaging;

Can be bought and sold any time during trading hours or even after hours;

Less volatile than stocks;

Supposedly more tax efficient than mutual funds;

Usually align themselves with some market index.

Leveraged ETF

Leveraged Exchange Traded Funds:
Attempt to enhance the return of an index (such as *NASDAQ* or $S\&P500$) by a factor ($2\times$ *or* $3\times$);
Invest borrowed money to do so;

$2\times$: When index goes up (down) by 1%, fund goes up (down) by 2%
$3\times$: When index goes up (down) by 1%, fund goes up (down) by 3%
$-1\times$: When index goes up (down) by 1%, fund goes down (up) by 1%
$-2\times$: When index goes up (down) by 1%, fund goes down (up) by 2%
$-3\times$: When index goes up (down) by 1%, fund goes down (up) by 3%

Last three choices correspond to *inverse leveraged funds*, and can be an alternative to *shorting*.

Leveraged ETF

Clearly, Leveraged ETFs are more volatile than index.

Enhanced returns by a specified factor *on a daily basis only*.

Due to rebalancing of assets on daily basis, use of borrowed money, and effect of compounding, a $2\times$ fund may not have two times return in long term.

In fact, in an static market going nowhere, one is bound to lose money in long run if invested in an leveraged fund.

Our Work

Designing of a leveraged exchange traded fund which essentially gives the same **average** return as a fund with specified leverage but has **less variability** (of the distribution of returns)and is **less volatile** (that is, less day-to-day variability of returns).

Our Work

The basic idea is:

Replace the daily rebalancing requirement by having a rule that when the market is up, sell $c\%$ of assets under management and when the market is down buy additional $c\%$ of assets under management. (c must be chosen carefully)

Let the leverage factor ($2\times$ or $3\times$) be random rather than fixed. In fact, even when leverage factor was targeted to be kept fixed at $2\times$ or $3\times$, we can never keep it fixed. Daily returns are assumed to have a probability distribution.

This reduces the standard deviation of the distribution of returns as well as day-to-day volatility, yet keeps the leverage factor, on an average, same. Long term performance remains same.

Simulations

Validation of the Work is based on extensive simulations.
Randomness of market returns and randomness of leverage factors result in a time series of returns of leverage funds.
Performance is shown in the tables and pictures that follow.

Mean Compounded Returns

EXHIBIT 2

Mean ($N_{sim} = 5{,}000$) Compounded Returns of the Index Fund (C_t), 2× Constant Leveraged Fund (C_t^*), and 2× Variable Leveraged Fund (C_t^{**}) over One Week for Various Values of Hedging Demand Ratio c

	Index Fund	2× Constant Leveraged Fund	Hedging Demand Ratio (c) for 2× Variable Leveraged Fund									
			$c = 1\%$		$c = 2\%$		$c = 3\%$		$c = 4\%$		$c = 5\%$	
Day	Mean of C_t	Mean of C_t^*	Mean of x_t	Mean of C_t^{**}	Mean of x_t	Mean of C_t^{**}	Mean of x_t	Mean of C_t^{**}	Mean of x_t	Mean of C_t^{**}	Mean of x_t	Mean of C_t^{**}
1	0.005	0.010	2.000	0.010	2.000	0.010	2.000	0.010	2.000	0.010	2.000	0.010
2	0.010	0.021	1.993	0.020	1.996	0.020	1.998	0.020	2.000	0.020	2.003	0.021
3	0.015	0.032	1.986	0.030	1.991	0.031	1.996	0.031	2.001	0.031	2.005	0.031
4	0.020	0.041	1.981	0.039	1.988	0.040	1.995	0.040	2.002	0.041	2.009	0.041
5	0.025	0.052	1.974	0.050	1.983	0.050	1.993	0.051	2.002	0.051	2.011	0.052

Standard Deviation of Compounded Returns

EXHIBIT 3

Standard Deviation ($N_{sim} = 5{,}000$) of Compounded Returns of the Index Fund (C_t), 2× Constant Leveraged Fund (C_t^*), and 2× Variable Leveraged Fund (C_t^{**}) over One Week for Various Values of Hedging Demand Ratio c

	Index Fund	2× Constant Leveraged Fund	Hedging Demand Ratio (c) for 2× Variable Leveraged Fund									
			$c = 1\%$		$c = 2\%$		$c = 3\%$		$c = 4\%$		$c = 5\%$	
Day	STD of C_t	STD of C_t^*	Mean of x_t	STD of C_t^{**}	Mean of x_t	STD of C_t^{**}	Mean of x_t	STD of C_t^{**}	Mean of x_t	STD of C_t^{**}	Mean of x_t	STD of C_t^{**}
1	0.015	0.030	2.000	0.029	2.000	0.029	2.000	0.029	2.000	0.029	2.000	0.029
2	0.021	0.061	1.993	0.042	1.996	0.042	1.998	0.042	2.000	0.042	2.003	0.042
3	0.026	0.069	1.987	0.053	1.991	0.053	1.996	0.053	2.001	0.053	2.005	0.053
4	0.030	0.076	1.981	0.061	1.988	0.061	1.995	0.062	2.002	0.062	2.009	0.062
5	0.034	0.083	1.974	0.069	1.983	0.069	1.993	0.069	2.002	0.070	2.011	0.070

Mean Compounded Returns: Longer Term

EXHIBIT 4

Mean (N_{sim} = 1,000) Compounded Returns of the Index Fund(C_t), 2× Constant Leveraged Fund (C_t^*), and 2× Variable Leveraged Fund (C_t^{**}) over Three Weeks (15 days) for Various Values of Hedging Demand Ratio c

	Index Fund	2× Constant Leveraged Fund	Hedging Demand Ratio (c) for 2× Variable Leveraged Fund									
			c = 1%		c = 2%		c = 3%		c = 4%		c = 5%	
Day	Mean of C_t	Mean of C_t^*	Mean of x_t	Mean of C_t^{**}	Mean of x_t	Mean of C_t^{**}	Mean of x_t	Mean of C_t^{**}	Mean of x_t	Mean of C_t^{**}	Mean of x_t	Mean of C_t^{**}
1	0.005	0.007	2.000	0.011	2.000	0.011	2.000	0.011	2.000	0.011	2.000	0.011
2	0.011	0.023	1.993	0.022	1.995	0.022	1.998	0.022	2.000	0.022	2.003	0.022
3	0.015	0.031	1.987	0.030	1.991	0.030	1.996	0.030	2.000	0.030	2.004	0.031
4	0.019	0.040	1.982	0.037	1.988	0.038	1.994	0.038	2.001	0.039	2.007	0.039
5	0.024	0.049	1.977	0.046	1.985	0.047	1.993	0.047	2.001	0.048	2.009	0.048
6	0.029	0.060	1.969	0.057	1.980	0.058	1.990	0.059	2.001	0.059	2.011	0.060
7	0.034	0.071	1.963	0.067	1.976	0.068	1.988	0.069	2.001	0.070	2.013	0.071
8	0.040	0.082	1.957	0.077	1.972	0.078	1.987	0.080	2.002	0.081	2.017	0.082
9	0.044	0.091	1.952	0.086	1.969	0.088	1.986	0.089	2.003	0.090	2.020	0.092
10	0.049	0.102	1.946	0.096	1.965	0.098	1.984	0.099	2.004	0.101	2.023	0.103
11	0.055	0.114	1.939	0.107	1.961	0.109	1.982	0.111	2.003	0.113	2.024	0.115
12	0.060	0.125	1.933	0.118	1.957	0.120	1.980	0.122	2.003	0.124	2.026	0.127
13	0.065	0.136	1.928	0.128	1.953	0.130	1.979	0.133	2.004	0.135	2.029	0.138
14	0.071	0.147	1.922	0.138	1.950	0.141	1.978	0.144	2.006	0.146	2.033	0.149
15	0.075	0.156	1.918	0.146	1.948	0.149	1.977	0.152	2.006	0.155	2.035	0.158

Standard Deviation of Compounded Returns: Longer Term

EXHIBIT 5

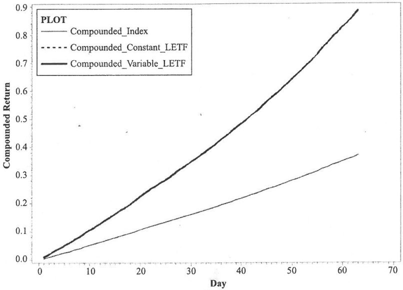
Standard Derivation ($N_{sim} = 1{,}000$) of Compounded Returns of the Index Fund (C_t), 2× Constant Leveraged Fund (C_t^*), and 2× Variable Leveraged Fund (C_t^{**}) over Three Weeks (15 days) for Various Values of Hedging Demand Ratio c

	Index Fund	2× Constant Leveraged Fund	Hedging Demand Ratio (c) for 2× Variable Leveraged Fund									
			$c = 1\%$		$c = 2\%$		$c = 3\%$		$c = 4\%$		$c = 5\%$	
Day	STD of C_t	STD of C_t^*	Mean of x_t	STD of C_t^{**}	Mean of x_t	STD of C_t^{**}	Mean of x_t	STD of C_t^{**}	Mean of x_t	STD of C_t^{**}	Mean of x_t	STD of C_t^{**}
1	0.015	0.030	2.000	0.030	2.000	0.030	2.000	0.030	2.000	0.030	2.000	0.030
2	0.021	0.060	1.993	0.043	1.995	0.043	1.998	0.043	2.000	0.043	2.003	0.043
3	0.027	0.070	1.987	0.055	1.991	0.055	1.996	0.055	2.000	0.055	2.004	0.055
4	0.031	0.076	1.982	0.062	1.988	0.063	1.994	0.063	2.001	0.063	2.007	0.063
5	0.035	0.083	1.977	0.070	1.985	0.071	1.993	0.071	2.001	0.071	2.009	0.072
6	0.039	0.091	1.969	0.078	1.980	0.079	1.990	0.079	2.001	0.080	2.011	0.080
7	0.042	0.097	1.963	0.085	1.976	0.085	1.988	0.086	2.001	0.086	2.013	0.087
8	0.044	0.101	1.957	0.089	1.972	0.089	1.987	0.090	2.002	0.090	2.017	0.091
9	0.046	0.106	1.952	0.094	1.969	0.094	1.986	0.095	2.003	0.096	2.020	0.097
10	0.049	0.112	1.946	0.100	1.965	0.101	1.984	0.102	2.004	0.103	2.023	0.104
11	0.053	0.120	1.939	0.106	1.961	0.108	1.982	0.109	2.003	0.110	2.024	0.111
12	0.055	0.125	1.933	0.111	1.957	0.113	1.980	0.114	2.003	0.116	2.026	0.117
13	0.057	0.131	1.928	0.116	1.953	0.118	1.979	0.119	2.004	0.121	2.029	0.123
14	0.059	0.136	1.922	0.120	1.950	0.122	1.978	0.124	2.006	0.126	2.033	0.128
15	0.063	0.143	1.918	0.127	1.948	0.129	1.977	0.131	2.006	0.134	2.035	0.136

Mean Compounded Returns

EXHIBIT 7

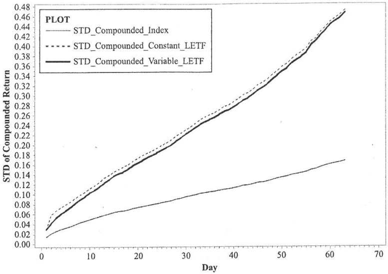
Mean (N_{sim} = 1,000) Compounded Returns of the Index Fund, 2× Constant Leveraged Fund, and 2× Variable Leveraged Fund over Three Months (63 days) with Hedging Demand Ratio c = 4% (last two curves largely overlap)



Standard Deviation of Compounded Returns

EXHIBIT 8

Standard Deviation (N_{sim} = 1,000) of Compounded Returns of the Index Fund, 2× Constant Leveraged Fund, and 2× Variable Leveraged Fund over Three Months (63 days) with Hedging Demand Ratio $c = 4\%$



Mean Volatility

AQ1 **EXHIBIT 13**

Mean Volatility ($N_{sim} = 1000$) of Compounded Returns for the Index Fund, {2×, 3×} Constant Leveraged Fund, and {2×, 3×} Variable Leveraged Fund over Different Time Periods for Various Values of Hedging Demand Ratio *c*

Target Leverage Multiple	Time Period	Index Fund	Constant Leverage	Hedging Demand Ratio (c) for Variable Leverage				
				$c = 1\%$	$c = 2\%$	$c = 3\%$	$c = 4\%$	$c = 5\%$
2	One Week (5 days)	0.025	0.063	0.050	0.050	0.050	0.050	0.050
	One Month (21 days)	0.050	0.116	0.102	0.104	0.105	0.107	0.109
	3 Months (63 days)	0.097	0.248	0.204	0.216	0.227	0.239	0.251
3	One Week (5 days)	0.025	0.096	0.075	0.075	0.075	0.075	0.075
	One Month (21 days)	0.050	0.185	0.153	0.155	0.156	0.158	0.159
	3 Months (63 days)	0.097	0.461	0.301	0.313	0.326	0.338	0.351