"Practice is the best of all instructors." - Unknown
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1. For the following functions:
    I. Find the domain of each of the following functions. State your answer in interval notation.
    II. Determine the value of x for which the function equals zero, i.e., the x-intercept.

a. $f(x) = \dfrac{2x - 3}{4x^2 - 16}$

b. $j(x) = \sqrt[3]{x}$

c. $p(x) = 2x^2 - 5x - 3$

d. $g(x) = \dfrac{2x - 4}{x^2 - 7x + 6}$

e. $m(x) = \sqrt{3x + 4}$

f. $r(x) = 5$

g. $F(x) = \dfrac{7}{\sqrt{5x - 3}}$

h. $K(x) = \dfrac{\sqrt{x}}{x - 5}$

i. $h(x) = \sqrt{x^2 - 25}$

j. $H(x) = \sqrt{\dfrac{x + 1}{x^2 + 5x + 6}}$

2. Simplify the following expressions. Assume that all variables represent positive numbers. Where applicable, write your final answer with positive exponents only.

a. $b\sqrt{9b^4c^3}$

b. $\sqrt[3]{64x^7y^{-5}}$

c. $\dfrac{\sqrt[4]{48x^9y^{13}}}{\sqrt[4]{3xy^{-3}}}$

d. $x^3 \cdot \sqrt[4]{x^8y^{-9}} \cdot \sqrt[4]{\dfrac{16x^{10}y^7}{27xy}}$

e. $\sqrt{\dfrac{16c^{16}}{3c^3d^2}}$

f. $\dfrac{8a}{\sqrt{8} + \sqrt{2}}$

g. $\sqrt[3]{\dfrac{54a^{11}}{2a^2b^6}}$

h. $\dfrac{4xy^2}{\sqrt[3]{8x^9y^6}}$

i. $\sqrt[3]{\dfrac{27a^3y^{12}}{100b^5x^3}}$

j. $\dfrac{-2 + \sqrt{3}}{2\sqrt{3} - 3\sqrt{2}}$

k. $\dfrac{\sqrt{x} - \sqrt{2}}{\sqrt{x} + \sqrt{2}}$

l. $\sqrt{\dfrac{a - b}{a + b}}$

m. $\left(\dfrac{16m^{10}n^4}{2m^{13}n^{-2}}\right)^{2/3}$

n. $\left(x^2y^{-1/3}z\right)^6 \left(x^{1/2}yz^{-2}\right)^2$

o. $\dfrac{12 - 15n}{5n^2 + 6n - 8}$

p. $\dfrac{\dfrac{12}{9a^2 - 4}}{\dfrac{16a^2}{3a^2 + 2a}}$

3. Solve each inequality. Where applicable, state your answer in interval notation. Where applicable, graph.

a. $x \leq 2$ and $2x + 3y \geq 3$

b. $y \leq \dfrac{1}{2}x$ or $2x + y \geq 2$

c. $x^3 + 3x^2 - 16x - 48 \geq 0$

d. $(6x + 5)^2 < (3x + 4)^2$

e. $\dfrac{(x + 2)^2}{x} < 0$

f. $\dfrac{3}{x - 2} \leq \dfrac{4}{x}$

4. Perform the operations and simplify. Assume that all variables represent positive numbers.

a. $\left(\sqrt{64}-6\right)^5$

b. $2\sqrt[3]{27x^3}+3\sqrt[3]{8x^3}-6\sqrt[3]{-x^3}$

c. $\sqrt[3]{-\dfrac{a^3}{27}+\dfrac{5a}{6}}$

d. $\left(\sqrt{32}+5\sqrt{8}-\sqrt{50}\right)^2$

e. $\left(2\sqrt{63}-5\sqrt{112}\right)^2$

f. $\sqrt{x^2}+7\sqrt{4x^2}-\sqrt{x^2-2x+1}$

g. $\left(\dfrac{9}{16}\right)^{1/2}+\left(\dfrac{1}{64}\right)^{5/6}$

h. $\left(\dfrac{1}{16}\right)^{-3/4}-\left(\dfrac{1}{49}\right)^{-1/2}$

i. $\dfrac{4x}{x^2-81}+\dfrac{3}{x^2-3x-54}$

j. $\dfrac{xy-4x+3y-12}{y^2-16}\div\dfrac{x^3+27}{6}$

k. $\dfrac{1}{x+y}+\dfrac{x}{x^2-y^2}-\dfrac{4}{2x-2y}$

l. $\left(\dfrac{p+1}{3p+4}\right)\left(\dfrac{1}{p+1}+2\right)$

m. $\dfrac{\dfrac{5}{h+2}+\dfrac{7}{2h-3}}{\dfrac{1}{h-3}+\dfrac{3}{2h-3}}$

n. $\dfrac{b+3}{5}+\dfrac{9-b^2}{5b+15}\div\dfrac{b-3}{b+3}$

5. Solve each equation for x.

a. $2x^2=5(x+3)$

b. $x+5\sqrt{x}-6=0$

c. $\sqrt{x+3}-\sqrt{x-1}=1$

d. $\sqrt[3]{6x+4}-4=0$

e. $\sqrt{x+\sqrt{6+x}}=\sqrt{2x}$

f. $x^{-2}+x^{-1}-30=0$
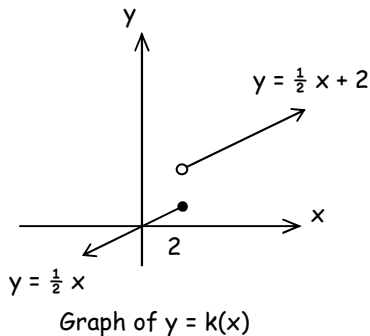
g. $\dfrac{8}{x+2}+\dfrac{x}{x+1}=\dfrac{5x+2}{x^2+3x+2}$

h. $\dfrac{5}{x+4}-\dfrac{2}{x}=-1$

i. $(3x+4)(x-2)=-9$

j. $r=\dfrac{ky}{x-az}$

6. Use the functions below to find the values of the functions in parts a – j. If no value can be found, state why.

$f(x)=-6x-2$

$g(x)=x^2-4x+1$

$h(x)=\dfrac{x}{x^2+3}$

$j(x)=\begin{cases} x^2+3 & \text{if } -2\le x\le 0 \\ -x+3 & \text{if } 0<x<3 \\ |2x-3| & \text{if } x\ge 3 \end{cases}$

Graph of $y=k(x)$

$y=\tfrac{1}{2}x+2$

$y=\tfrac{1}{2}x$

a. $h\left(\sqrt{3}\right)$

b. $f(7x+4)$

c. $g(x+2)$

d. $g\left(2+\sqrt{3}\right)$

e. $j(-2)$
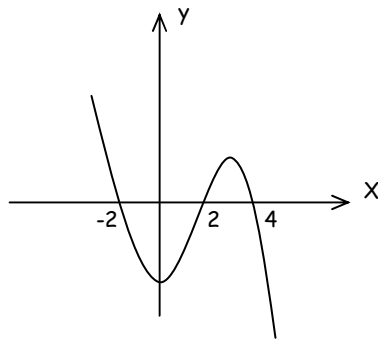
f. $j(2)$

g. $j(3)$

h. $j(16)$

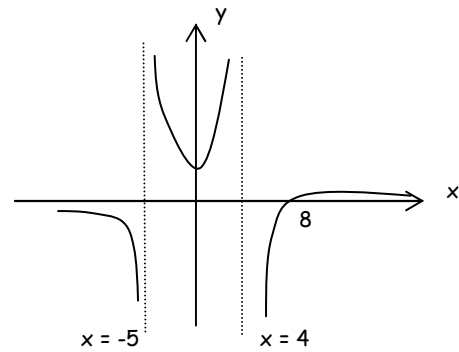i. $g(1)-f(1)$

j. $h(0)-g(2)$

k. $k(18)$

l. $k(-8)$

7. Solve the indicated inequalities using the graphs.

   a. $f(x) > 0$, $f(x) \leq 0$ b. $g(x) \geq 0$, $g(x) < 0$

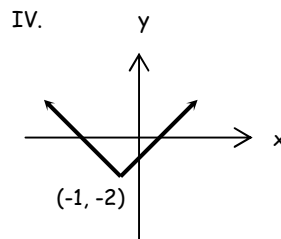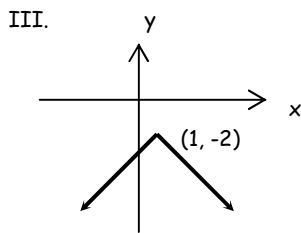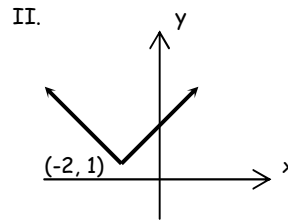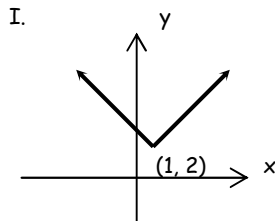Graph of y = f(x)   Graph of y = g(x)

8. Graph the piecewise function $g(x) = \begin{cases} -x + 2 & \text{if} \quad x < 3 \\ 2x - 4 & \text{if} \quad x \geq 3 \end{cases}$.

9. Write each quadratic function in the form $y = a(x - h)^2 + k$ by <u>completing the square</u>. Then graph the function. Include the vertex, intercepts and axis of symmetry on your graph. Show all your work.

  a. $y = -2x^2 - 8x + 2$ b. $y = \frac{1}{2} x^2 - 4x + 9$

10. Match <u>one</u> function to each graph.

  I.   II.

                 (1, 2)   (-2, 1)

a. $y = |x - 2| - 1$

b. $y = |x + 2| - 1$

c. $y = -|x - 1| - 2$

d. $y = |x - 1| + 2$

  III.   IV.

e. $y = -|x + 1| + 2$

         (1, -2)

f. $y = |x + 2| + 1$

                               (-1, -2)

g. $y = -|x + 1| - 2$

h.   None of the above

11. If the following transformations are performed on the graph of f(x) to obtain the graph of g(x), write the equation of g(x).

a. $f(x) = x^2$ is shifted right 5 units and down 4 units.

b. $f(x) = \sqrt{x}$ is shifted left 3 units, reflected over the x-axis, and up 8 units.

12. Find the number and type of solutions of the equation $3x = 1 + 5x^2$. Do not solve.

13. Flying at a constant speed, a plane can travel 800 miles with the wind in the same amount of time it can fly 650 miles against the wind. If the wind blows at 30 mph, what is the speed of the plane?

14. If Brian can paint his room in 5 hours, but his brother could paint the room on his own in 4 hours, how long would it take for the two of them to paint the room together?

15. An object is thrown upward from a height of 240 feet so that its height h (in feet) t seconds after being thrown is given by

$$H(t) = -16t^2 + 32t + 240$$

a. How long does it take the object to reach its maximum height?

b. What is the maximum height attained by the object?

c. How long does it take the object to hit the ground?

16. For parts a and b, it may help to sketch the parabola using the given information.

a. A parabola opens upward, has a vertex at (4, -6), and an x-intercept at (1, 0). Find the other x-intercept.

b. A parabola passes through the point (8, 9) and has a vertex at (4, 3). The point (u, 9) also lies on the parabola. Find the value of u.

17. A jewelry maker sells rings at craft shows. The profit (in dollars) depends on the number of rings produced, x, according to

$$p(x) = -\frac{1}{10}x^2 + 42x - 1260$$

a. How much profit does the jeweler make when 10 rings are produced?

b. How many rings must be produced for the jeweler to break even? Round to the nearest whole unit.

c. Find the vertex.

d. Find the number of rings for which the profit is maximized.

e. What is the maximum profit?

18. Pat has two finals back to back on the same day: one in math, and the other in biology. Pat has a maximum of 12 hours of study time before the exams, and must divide the study time between the two subjects. Let x represent the number of hours Pat spends studying math, and let y represent the number of hours Pat spends studying biology.

a. Find a set of inequalities to describe the constraints on Pat's study time.

b. Graph the constraints to find the feasible region defining Pat's study time.

c. Does the point (4, 9) lie in the feasible region?

19. Find the dimensions of the rectangular garden of greatest area that can be enclosed with 40 feet of fencing.