MTH 141 Final Exam Name____________________

April 20, 2012

Directions: Show all work for full credit. Books and notes are not allowed on this exam. You may use a calculator, but if a problem asks for an exact answer, you may not use a calculator. If you are unclear about anything, ask.

Turn off cell phones and electronic communication devices and stow them out of view. (If there is a potential emergency for which you need access to a cell phone or such, let us know at the beginning of the exam so you can be appropriately seated. Some people in the room have text alert and will have their cell phones on in case of an emergency.)

Circle the item below that identifies your workshop section:

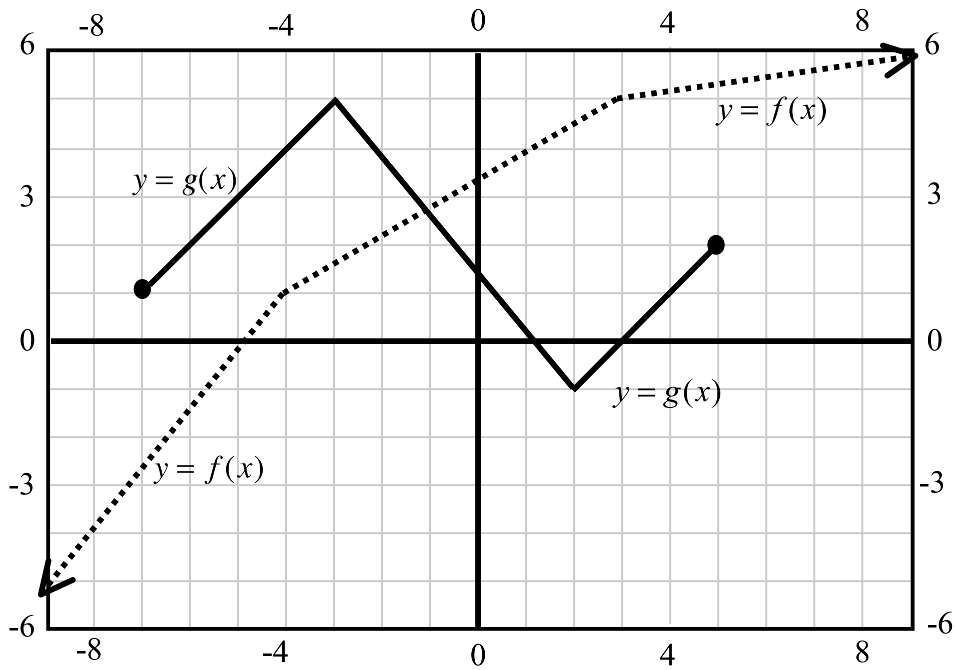
Bailey 10:40-11:47 MWF	Dambrun 12:00-1:07 MWF	Burke 1:20-2:27 MWF	Mullally 1:20-2:27 MWF
Dambrun 2:0-3:47 MWF	Pate 8:00-9:47 TuTh	Pate 10:00-11:47 TuTh	

Scoring:

1.___________	7.___________	13.__________
2.___________	8.___________	14.__________
3.___________	9.___________	15.__________
4.___________	10.__________	16.__________
5.___________	11.__________	17.__________
6.___________	12.__________	EC__________

Total________/250___

1. [3 pts each] The graphs of the functions *f* and *g* are in the sketch shown below. The graph of *f* is dashed so that you can easily discern which is which. Use the sketch to answer the following questions. Estimate if necessary.



(a) Evaluate $(gf)(0)$ ____________

(b) Solve the inequality $f(x) > 0$. ____________

(c) Give the domain of g. Use interval notation. ____________

(d) Give the range of g. Use interval notation. ____________

(e) Evaluate $(f \circ g)(1)$. ____________

(f) Evaluate $f^{-1}(4)$ ____________

2. [2 pts each] Give <u>exact values</u> of each of the following. If an item is not defined, write "undefined."

$\sec 45^\circ$ ____________ $\qquad$ $\sec^{-1} 2$ ____________

$\log_5 \frac{1}{\sqrt{5}}$ ____________ $\qquad$ $\sin^{-1} 2$ ____________

$\ln e^{\pi}$ ______________ $\qquad$ $W(\pi)$ ____________

3. [3 pts each] Fill in the missing blank in each of the following.

$(\sin x + \cos x)^2 = 1$ for every real number x is _________________ (true or false)

If x is a positive real number, then $\log_5 x$ is the number t for which ____________

In solving the equation $\cos x = -\frac{1}{2}$, $\cos^{-1}(1/2)$ is the ___________ of all of the solutions.

The reference angle for –223° is __________.

$2\ln x - 3\ln y$ can be expressed as a single logarithm as ________________.

4. [8 pts] Find the domain of the function $g(x)=\sqrt{\dfrac{x-3}{(x-7)(x+4)}}$. Put your answer in interval form.

5. [10 pts] Let $a=\ln x, b=\ln(x+3),$ and $c=\ln(x-2)$. Express $4+\ln\left(\dfrac{x\sqrt{x-2}}{e^3(x+3)}\right)$ in terms of *a*, *b*, and *c* only. (The final expression should not involve *e*.)

6. [5 pts each] Let. $f(x)=1-\ln(8-2x)$

(a) Find the x-intercept(s) of f. Exact answers please.

(b) Find the y-intercept(s) of f. Exact answers please.

(c) Find the domain of f. Express it in interval form.

7. [5 pts each] Let $f(x) = 8\cos(6x - \pi)$.

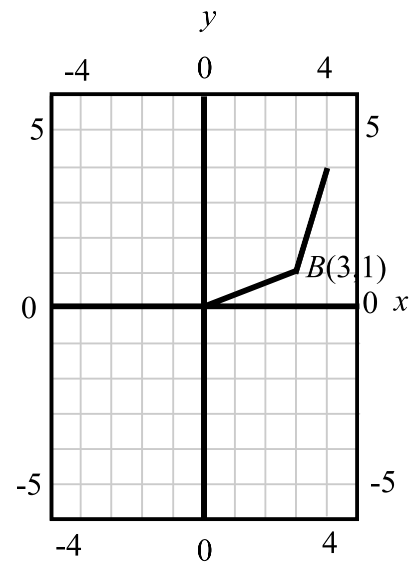
(a) Find the amplitude of *f*.

(b) Find the period of *f*.

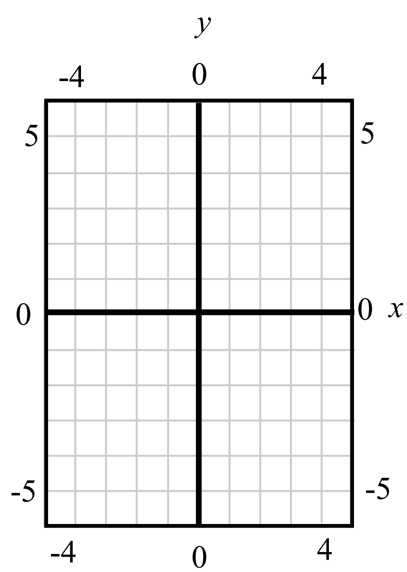
(c) Describe the means by which the graph of *f* is obtained from the graph of $y = 8\cos(6x)$. Your answer should be of the form "Shift $y = 8\cos(6x)$ so many units (right, left, up, down).

(d) Find the range of the function $g(x) = 2 + 8\cos(6x - \pi)$

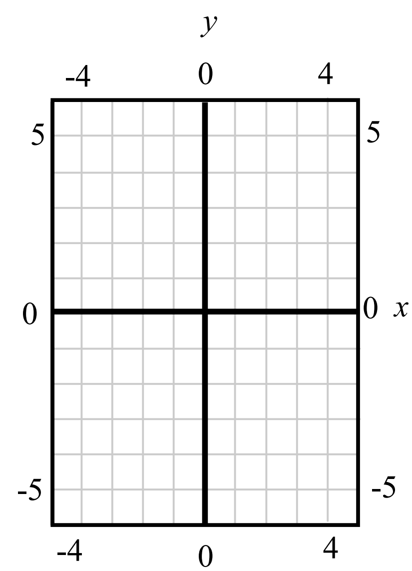
8. [5 pts each] The graph of a function h is shown to the right. In each of the parts below, carefully sketch the indicated function. In each part, label the coordinates of the point that corresponds to the "corner point" B in the graph of h.



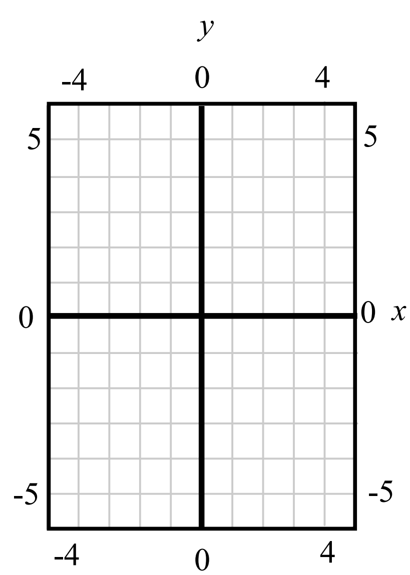
(a) $y = h(x+1) - 2$



(b) $y = h(-x)$



(c) $y = h^{-1}(x)$



9. [9 pts each] Find all solutions of each of the following equations. Your solutions must be exact, and a calculator solution with no work or an approximation is not adequate.

(a) $5+3^{2x-1}=16$

(b) $\sin^2 x+3=3\cos x$

(c) $4=3+2\ln(x+1)$

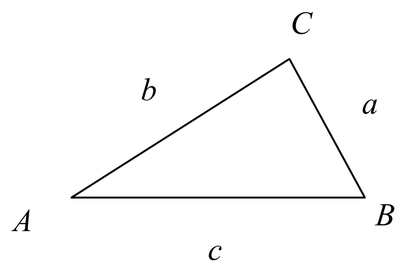
10. [10 pts] Find all solutions of the equation below that are in the interval $[0^\circ, 360^\circ)$. Degree measure is being used here, and exact solutions are required.

$$2\cos(3x) + 1 = 0$$

11. [10 pts] Let $g(x) = x^2 - 2x$. Evaluate and simplify the following quotient:

$$\frac{g(t+h) - g(t)}{h}$$

12. [9 pts] You are given that the sides of a triangle are $a = 6, b = 10, c = 13$ where the angles and sides of the triangle follow the convention shown. Find the angle C to the nearest hundredth of a degree.



13. [9, 5 pts] An airplane is flying directly between the cities of Alzeda and Busby which are exactly 50 miles apart. An observer in spots the plane, and her angle of elevation to the plane is 10°. At the very same time, an observer in Busby has an angle of elevation of 15° to the plane.

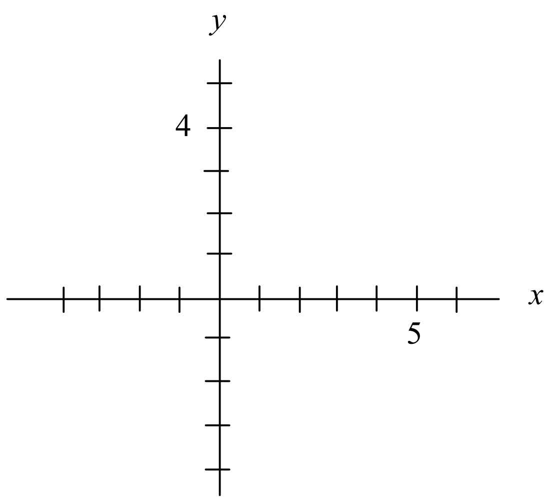
(a) Find the distance of the plane to Alzeda. (Note: This is not the distance along the ground. Round your answer to 2 decimal places.)

(b) Find the plane's elevation.

14. [6, 8 pts] (a) A rational function *f* has *x*-intercepts $(0,0)$ and $(2,0)$ and satisfies the following:

as $x \to \infty$, $f(x) \to 2$, and as $x \to -\infty$, $f(x) \to 2$
as $x \to 1^-$,, $f(x) \to -\infty$, and as $x \to 1^+$, $f(x) \to -\infty$

Sketch the graph of f on the axes below. Be sure to also graph any asymptotes of *f* with dotted lines.



(b) Find a valid equation for $f(x)$.

15. [4, 8, 3 pts resp.] Let $P(x) = 5x^3 - 9x^2 + 8x + 2$

(a) Write the candidates for the rational zeros of $P(x)$ according to the Rational Zero Theorem.

(b) Find all zeros of $P(x)$. Exact answers are required, and complete work must be shown. Solution by advanced calculator is not adequate.

(c) Factor $P(x)$ completely into linear factors.

16. [10 pts each] Prove each of the following equation is an identities. Be aware that your proof must be logically correct. You are also expected to write the justification for each step (name of the identity used or "algebra.").

(a) $\dfrac{\sin(x-y)}{\sin x \sin y} = \cot y - \cot x$

(b) $\cos(3\theta) = 4\cos^3\theta - 3\cos\theta$

17. [8, 10 pts] Give an exact value for each of the following:

(a) $\cos^{-1}(\cos(19\pi/15))$

(b) $\cos\left[\sin^{-1}(3/5)-\pi/3\right]$

Bonus [5 pts]: Explain why the following statement is true. "The sine function does not distinguish between acute and obtuse angles, but the cosine function does distinguish between acute and obtuse angles."

Bonus [3 pts]: State what is used in deriving the Law of Sines. You do not need to derive the Law of Sines. Just give the main item used in its derivation.

Bonus: [2 pts] In the text book, every example is followed by a ______________. (Fill in the blank what the text calls these items.)

Some Trigonometric Identities and the like

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Sum and Difference Identities

$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$

$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$

$\tan(x \pm y) = \dfrac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$

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Double-Angle Identities

$\sin 2x = 2 \sin x \cos x$

$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$

$\tan 2x = \dfrac{2 \tan x}{1 - \tan^2 x}$

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Half-Angle Identities

$\sin \frac{x}{2} = \pm\sqrt{\dfrac{1 - \cos x}{2}}$

$\cos \frac{x}{2} = \pm\sqrt{\dfrac{1 + \cos x}{2}}$

$\tan \frac{x}{2} = \dfrac{\sin x}{1 + \cos x} = \dfrac{1 - \cos x}{\sin x}$

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Product to Sum Identities

$\sin x \cos y = \frac{1}{2}[\sin(x+y) + \sin(x-y)]$

$\cos x \sin y = \frac{1}{2}[\sin(x+y) - \sin(x-y)]$

$\cos x \cos y = \frac{1}{2}[\cos(x-y) + \cos(x+y)]$

$\sin x \sin y = \frac{1}{2}[\cos(x-y) - \cos(x+y)]$

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Sum to Product Identities

$\sin x + \sin y = 2\sin\left(\dfrac{x+y}{2}\right)\cos\left(\dfrac{x-y}{2}\right)$

$\sin x - \sin y = 2\cos\left(\dfrac{x+y}{2}\right)\sin\left(\dfrac{x-y}{2}\right)$

$\cos x + \cos y = 2\cos\left(\dfrac{x+y}{2}\right)\cos\left(\dfrac{x-y}{2}\right)$

$\cos x - \cos y = -2\sin\left(\dfrac{x+y}{2}\right)\sin\left(\dfrac{x-y}{2}\right)$

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Formulas for arcs, areas of sectors, angular speed and linear speed

$s = r\theta$

$A = \frac{1}{2}r^2\theta$

$\omega = \theta / t$

$v = r\omega$

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Law of Sines

$\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$

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Law of Cosines

$a^2 = b^2 + c^2 - 2bc\cos A$

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