COLLOQUIUM

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Number of Solutions of Equations Over Finite Fields and Coding Theory

Abstract

We consider the problem of determining the maximum number of common solutions of a bunch of polynomials over a finite field. The simplest case is of course of a single (nonzero) polynomial in one variable, where the degree usually gives the maximum number of solutions. In the general case of several polynomials in several variables, the problem is meaningful and interesting when the base field is finite and the solutions are sought in the corresponding affine or projective space over the given finite field. When these polynomials are assumed linearly independent and of a degree bounded by a fixed positive integer, the problem is equivalent to a problem in coding theory, namely, that of determining the generalized Hamming weights of Reed-Muller codes. The known solution in this case, due to Heijnen and Pellikaan (1998) uses results in combinatorics such as the Kruskal-Katona theorem.

The case of systems of linearly independent multivariate homogeneous polynomials, all of the same degree, where the zeros are considered in a projective space over the given finite field is perhaps even more interesting. There is an elaborate conjecture of Tsfasman and Boguslavsky that predicts the maximum value when the degree of the homogeneous polynomials is not too large in comparison to the size of the finite field. Special cases of the conjecture are known to be true, thanks to the results of Serre (1991) and Boguslavsky (1997), but the general case has been open for quite some time.

We will give a motivated account of the above problem and its alternative formulations while briefly explaining the relevant background. We will then describe some recent developments that has led to significant new results concerning the general case.

Joint work with Mrinmoy Datta.

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3 – 4 P.M.
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(Refreshments at 2:30-3:00 PM in the kitchen area adjacent to 368 SEB)